Table 2

$\dot{H}_x\vert_0 = F$	$-k(\partial\theta/\partial x)_0 = F$
$\theta(x = 0) = 1.068(F/c)(t/\alpha)^{1/2}$ $-k(\partial\theta/\partial x)\vert_0 = 0.761\ F$ Energy content $= Ft$	$\theta(x = 0) = 1.32(F/c)(t/\alpha)^{1/2}$ $-k(\partial\theta/\partial x)\vert_0 = F$ Energy content $= \frac{7}{6}\ Ft$
Exact solution	
$T(x = 0) = 1.13\ (Fc)(t/\alpha)^{1/2}$ $-\ k(d\theta/dx\vert_0 = F$	

of which to use depends upon the use to which the solution will be put.

These methods correspond to the collocation methods commonly employed in solid mechanics where one either satisfies the field equations exactly and the boundary conditions approximately or satisfies the boundary conditions exactly and the field equations approximately. Since the limitations of the collocation methods are well known to those in applied mechanics, but because most workers in heat conduction are unfamiliar with the differing results of the two methods, it is of value to indicate the differences.

To illustrate the point, consider a semi-infinite region that at time zero has a constant heat flux F applied to its surface. We may choose the temperature profile to be $\theta = q_1[1 - (x/q_2)]^2$ where θ is the temperature measured above the initial value, q_1 represents the surface temperature, x is the distance from the surface, and q_2 is the penetration depth. The boundary conditions are

$$-k(\partial\theta/\partial x)|_{x=0} = F \qquad \text{and} \qquad k(\partial\theta/\partial x)|_{x=q_2} = 0$$

Now the heat-flux vector $\mathbf{H}$, defined by $\operatorname{div}\mathbf{H} = -c\theta$, reduces to the single component H_x and is given by[1]

$$H_x = (cq_1q_2/3)[1 - (x/q_2)]^3$$

The generalized coordinates are related now by either the flux boundary condition or by the total energy balance given by the relation[3]

$$\text{energy addition rate} = \int_S \frac{\partial H_x}{\partial t}|_0\, d\sigma = \dot{H}_x|_0 A = FA$$

Table 1 indicates the pertinent results. Even though the thermal potential and dissipation function are quite different, the penetration depths are in good agreement. However, when one considers the surface heat flux and total energy content one obtains results as shown in Table 2. It is seen that these values are greatly different and that either the boundary condition or the energy content are noticeably in error. Obviously, if the purpose of the calculation were to compute the energy loss in a surrounding media, or to evaluate the average stress in the region, the heat balance must be satisfied. On the other hand if the gradient at the surface is of importance, e.g., for the calculation of the gradient of thermal stresses for crack propagation, then the exact boundary condition must be used. Either case may be used if only the surface temperature is desired. Consequently one must exercise care in the method used to determine those generalized coordinates that pertain to the surface conditions.

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Modification of Magnetic Energy by Differential Fluid Motions

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I. Introduction

As the solar plasma moves up in the solar atmosphere to form solar wind, many important physical phenomena are produced by the interaction between the solar plasma and the magnetic field encountered by the solar plasma. The interplanetary magnetic field is believed to originate from the general field of the sun, for example, by the convective motion of the solar wind against the general field.[1]

A basic question is how a magnetic field is modified by the motion of an electrically conducting fluid when the kinetic energy of the flow is well above the magnetic energy. Clark[2] has studied this problem by considering a unidirectional magnetic field $H(y, t)$ acted upon by a two-dimensional stagnation flow given by

$$u = Ax \qquad v = -Ay \tag{1}$$

where x, y are Cartesian coordinates; u, v are the x and y velocity components, respectively; and A is a positive constant. He found that the flow tends to increase the magnetic energy by a stretching of the field lines and to increase the rate of dissipation through Joule heating.

The purpose of this work is to show that the same flow can act, however, to diminish the magnetic energy, if the unidirectional magnetic field is rotated 90° in the x-y plane (Fig. 1) such that

$$H_x = 0 \qquad H_y = H(x, t) \tag{2}$$

Diminution, instead of amplification, of the magnetic energy can be expected, since the flow acts to expand the lines of force.

II. Solutions and Discussions

By using Eqs. (1) and (2), the magnetohydrodynamic equations[2,3] become

$$\partial H/\partial t + A(\partial/\partial x)(xH) = \eta(\partial^2 H/\partial x^2) \tag{3}$$

where η is the magnetic diffusivity. This equation differs from the basic equation, Eq. (7), in Clark's paper by the sign in the second term of the left-hand side. It follows immediately that the total magnetic energy E and the total rate of dissipation D defined by

$$E(t) = \frac{\mu}{8\pi}\int_{-\infty}^{\infty} [H(x, t)]^2 dx$$

and

$$D(t) = \frac{\mu\eta}{4\pi}\int_{-\infty}^{\infty} \left[\frac{\partial H(x, t)}{\partial x}\right]^2 dx$$

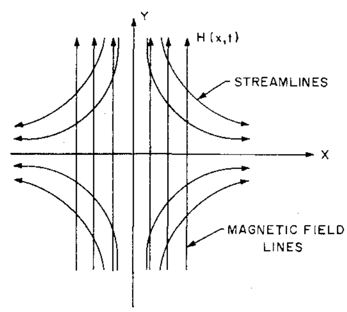

Fig. 1 Streamlines of stagnation flow and magnetic field lines.

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satisfy the equation

$$dE/dt = -AE - D \tag{4}$$

Thus, the magnetic energy is diminished by the fluid motion of the form given by Eq. (1). It is to be noted that essentially the same result will be obtained if in Clark's work the sign of A is taken to be negative so that the flow direction is reversed.

Equation (3) can be reduced to the ordinary diffusion equation $\psi_\tau = \psi_{ss}$ by the transformation $\psi = H\exp(At)$, $\tau = (\eta/A)\{1 - \exp(-2At)\}$ and $s = x\exp(-At)$. Thus, the solution of Eq. (3) can be expressed in terms of the initial profile of the magnetic field $H_0(x)$ as follows:

$$H(x, t) = (2\pi\beta)^{-1/2}e^{-At}\int_{-\infty}^{\infty} H_0(\xi)\exp\frac{-(\xi - xe^{-At})^2}{2\beta}\,d\xi \tag{5}$$

where $\beta = (\eta/A)(1 - e^{-2At})$. Three examples have been worked out in detail for initial profiles:

$$H_0(x) = h_0\exp(-x^2/2\delta^2) \tag{6}$$

$$H_0(x) = (h_0x/\delta)\exp(-x^2/2\delta^2) \tag{7}$$

$$H_0(x) = h_0(x/\delta)^2\exp(-x^2/2\delta^2) \tag{8}$$

where h_0 and δ are constants. The solutions are, respectively,

$$H(x, t) = h_0\lambda^{-1/2}\exp\left\{-At - \frac{1}{2\lambda}\left(\frac{x}{\delta}\right)^2 e^{-2At}\right\} \tag{9}$$

$$H(x, t) = \left(\frac{h_0x}{\delta}\right)\lambda^{-3/2}\exp\left\{-2At - \frac{1}{2\lambda}\left(\frac{x}{\delta}\right)^2 e^{-2At}\right\} \tag{10}$$

$$H(x, t) = h_0\lambda^{-5/2}\left\{(\lambda - 1)\lambda e^{2At} + \frac{x^2}{\delta}\right\}\exp\left\{-3At - \frac{1}{2\lambda}\left(\frac{x}{\delta}\right)^2 e^{-2At}\right\} \tag{11}$$

where

$$\lambda = 1 + \epsilon(1 - e^{-2At}) \qquad \epsilon = \eta/A\delta^2$$

The total magnetic energy and the rate of dissipation are of the form, respectively,

$$E \sim \lambda^{-1/2}e^{-At} \qquad E \sim \lambda^{-3/2}e^{-At} \qquad E \sim \lambda^{-5/2}e^{-At}$$

$$D \sim \lambda^{-3/2}e^{-3At} \qquad D \sim \lambda^{-5/2}e^{-3At} \qquad D \sim \lambda^{-7/2}e^{-3At}$$

Since λ varies from 1 to $1 + \epsilon$ as t goes from 0 to infinity, the variation will be small for $\epsilon \ll 1$. The difference between these expressions for the three cases is, therefore, only slight.

Examination of the solutions obtained shows that the magnetic field intensity does, however, go up in value for $(x/\delta)^2$ sufficiently large, namely, for

$$x^2/\delta^2 > \lambda e^{2At} \tag{12}$$

Since the diminishing of field intensity at other values of x is large, the total magnetic energy actually goes down for all t. For the first example given by Clark [corresponding to Eq. (7)], it can be shown that

$$\begin{array}{lll} \partial H/\partial t \gtrless 0 & \text{when } (y^2/\delta) \lessgtr \epsilon + 2(1-\epsilon)e^{-2At} - \epsilon^2(1-\epsilon)^{-1}e^{2At} & t < t_E \\ = 0 & \text{at } \quad y = 0 & \\ < 0 & \text{at } \quad y \neq 0 & \left.\right\} \text{when } t = t_E \\ < 0 & \text{for all } y & \text{when } t > t_E \end{array}$$

where $t_E = (1/2A)\ln\{2(1 - \epsilon)/\epsilon\}$.

The foregoing results show how the magnetic field energy will be modified by a stagnation flow. This modification depends on the orientation of the field with respect to the flow. In the three preceding examples for both symmetrical and antisymmetrical field profiles, the stagnation flow acts to lessen the total magnetic energy. But if the field direction is rotated 90° in the plane of the flow, amplification of the total magnetic energy will occur as shown by Clark.

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Indirect Speed Measurement for Planetary Entry

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Nomenclature

- A = area
- C_D = drag coefficient
- h = altitude
- m = mass
- $\dot{q}$ = heating rate/ft²
- V = velocity
- β = inverse scale height
- θ = flight-path angle
- ρ = density
- ρ_0 = surface density

Subscripts

- E = at entry
- dep = at deployment

Introduction

UNMANNED capsules, which enter planetary atmospheres at hypersonic velocities must be able to sense when the velocity has been reduced to a safe value if a parachute is to be deployed. Maximum time to sample the atmosphere and acquire experimental data on the atmospheric temperature, density, and composition profiles will be afforded by deploying a parachute at supersonic velocities. Two conditions must prevail at the time of parachute deployment in order to assure successful operation. First, the dynamic pressure must be low enough so that the parachute and shroud-line stresses do not exceed design limits. Second, if the parachute is deployed at supersonic velocities, aerodynamic heating must be below critical values. Unfortunately, direct measurement of the density and velocity behind the shock with sufficient accuracy is difficult, and inertial measurement or radar altimetry generally requires heavy, complex instrumentation.

This paper describes a simple technique for sensing when the velocity of a ballistic capsule entering a planetary atmosphere has decreased sufficiently to permit a parachute to be opened safely. The proposed technique does not require integration of the deceleration profile; rather, the velocity and density criteria are transformed to an acceleration criterion, which can be sensed easily with a simple accelerometer.

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